Emerging Universe from Scale Invariance

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Abstract

We consider a scale invariant model which includes a R^2 term in action and show that a stable "emerging universe" scenario is possible. The model belongs to the general class of theories, where an integration measure independent of the metric is introduced. To implement scale invariance (S.I.), a dilaton field is introduced. The integration of the equations of motion associated with the new measure gives rise to the spontaneous symmetry breaking (S.S.B) of S.I. After S.S.B. of S.I. in the model with the R^2 term (and first order formalism applied), it is found that a non trivial potential for the dilaton is generated. The dynamics of the scalar field becomes non linear and these non linearities are instrumental in the stability of some of the emerging universe solutions, which exists for a parameter range of the theory.

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I. INTRODUCTION

In modern cosmology our notions concerning the early universe have introduced a new element, the inflationary phase of the early universe [1], which provides an attractive scenario for solving some of the fundamental puzzles of the standard Big Bang model, like the horizon and the flatness problems as well as providing a framework for sensible calculations of primordial density perturbations.

Even in the context of the inflationary scenario however one encounters the initial singularity problem which remains unsolved, showing that the universe necessarily had a beginning for generic inflationary cosmological model[2].

Recently models that can avoid those conclusions have been discovered [3–9]. The way to escape the singularity in these models is to violate the geometrical assumptions of these theorems, which assume i) that the universe has open space sections ii) the Hubble expansion is always greater than zero in the past. In [3],[4] the open space section condition is violated since closed Robertson Walker universes with $k = 1$ are considered and the Hubble expansion can become zero, so that both i) and ii) are avoided.

In [3] even models based on standard General Relativity, ordinary matter and minimally coupled scalar fields were considered and can provide indeed a non singular (geodesically complete) inflationary universe, with a past eternal Einstein static Universe that eventually evolves into an inflationary Universe.

Those most simple models suffer however from instabilities, associated with the instability of the Einstein static universe. The instability is possible to cure by going away from GR, considering non perturbative corrections to the Einstein‘s field equations in the context of the loop quantum gravity[5], a brane world cosmology with a time like extra dimension[6], considering the Starobinski model for radiative corrections (which cannot be derived from an effective action)[7], exotic matter[8] or $f(R)$ theories in the presence of perfect fluids with $w < 0$ [9]. In addition to this, the consideration of a Jordan Brans Dicke model also can provide a stable initial state for the emerging universe scenario [10].

In this paper we propose a different theoretical framework where such emerging universe scenario is realized in a natural way, where instabilities are avoided and a successful inflationary phase with a graceful exit can be achieved.

We work in the context of a two measures theory (TMT) [11] and more specifically in

the context of the scale invariant realization of such theories [12–15]. These theories can provide a new approach to the cosmological constant problem and can be generalized to obtain also a theory with a dynamical spacetime [16] . We will show how the stated goals can be achieved in the framework of the TMT models.

The paper will be organized as follows: In section II the effective field Eqs. in the Einstein frame, give rise to an effective potential for a dilaton field (needed to implement a model with global scale invariance) which presents a flat region. This model can describe an emerging universe scenario. In section III, we look at the generalization of this model [15] by adding a curvature square, "R^2", and show that the resulting model contain a flat region. Furthermore the universe can evolve into an inflationary state which can undergo a successful graceful exit. We end with a discussion and conclusions section.

II. THE EMERGING SCENARIO AFTER THE INTRODUCTION OF A R^2 TERM

It is possible to obtain a model that through a spontaneous breaking of scale invariance can give us an emerging universe scenario. We consider the scale invariant action of the form [11, 12]

$$S_L = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x + \epsilon \int (g^{\mu\nu} R_{\mu\nu}(\Gamma))^2 \sqrt{-g} d^4x. \tag{1}$$

Here, L_1 and L_2 are given by

$$L_1 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi), \tag{2}$$

and

$$L_2 = U(\phi), \tag{3}$$

respectively. Also, Φ is defined as $\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$ where φ_a $(a = 1, 2, 3, 4)$ is a 4-scalar fields. The last term in the previous action is not only globally scale invariant, but also locally scale invariant, that is conformally invariant.

Let us see which are the equations of motion. The variation of the action respect to the 4-scalar fields, φ_a, yields to

$$A^\mu_a \partial_\mu L_1 = 0, \tag{4}$$

where $A^\mu_a = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d$, and since $det(A^\mu_a) = \frac{4^{-4}}{4!}\Phi^3$ we get $\partial_\mu L_1 = 0$, for $\Phi \neq 0$, and thus L_1 becomes

$$L_1 = \frac{-1}{\kappa}R(\Gamma, g) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V = M, \tag{5}$$

where M is an integration constant.

The variation of the action with respect to the metric $g^{\mu\nu}$ gives

$$R_{\mu\nu}(\Gamma)(\frac{-\Phi}{\kappa} + 2\epsilon R\sqrt{-g}) + \Phi\frac{1}{2}\phi_{,\mu}\,\phi_{,\nu} - \frac{1}{2}(\epsilon R^2 + U(\phi))\sqrt{-g}g_{\mu\nu} = 0. \tag{6}$$

It is interesting to notice that if we contract this equation with $g^{\mu\nu}$, the ϵ terms do not contribute. Solving the scalar curvature from this and inserting in the other ϵ - independent equation $L_1 = M$ we get the solution for the ratio of the measures i.e. $\chi = \frac{\Phi}{\sqrt{-g}} = \frac{2U(\phi)}{M+V(\phi)}$.

We see that under a conformal transformation given by

$$\overline{g}_{\mu\nu} = (\frac{\omega}{\sqrt{-g}})g_{\mu\nu} = (\chi - 2\kappa\epsilon R)g_{\mu\nu}, \tag{7}$$

the new metric $\overline{g}_{\mu\nu}$ defines the known Einstein frame. Equations (6) can now be expressed in this frame as

$$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{g}_{\mu\ \nu}\overline{R} = \frac{\kappa}{2}T^{eff}_{\mu\nu}, \tag{8}$$

where

$$T^{eff}_{\mu\nu} = \frac{\chi}{\chi - 2\kappa\epsilon R}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\overline{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\overline{g}^{\alpha\beta}) + \overline{g}_{\mu\nu}V_{eff}, \tag{9}$$

with

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} + \lambda. \tag{10}$$

The constant λ that appears in this latter expression could be obtained from the original lagrangian, where in it is added a term proportional to $\lambda\Omega^4$, where Ω is the conformal factor. This renders the action to be

$$S_{eff,\Lambda} = \int \sqrt{-\overline{g}}d^4x\left[-\frac{1}{\kappa}\overline{R}(\overline{g}) + p\left(\phi, R\right)\right], \tag{11}$$

where

$$p(\phi, R) = \frac{\chi}{\chi - 2\kappa\epsilon R}\frac{1}{2}\overline{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V_{eff}, \tag{12}$$

with V_{eff} given by Eq. (10).

Also, Eq.(5) expressed in terms of $\overline{g}^{\alpha\beta}$ becomes $\frac{-1}{\kappa}R(\Gamma,g)+(\chi-2\kappa\epsilon R)\frac{1}{2}\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V=M$, where we have written V in place of $V(\phi)$. This allows us to solve for R and thus we get

$$R=\frac{-\kappa(V+M)+\frac{\kappa}{2}\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\chi}{1+\kappa^2\epsilon\overline{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}. \tag{13}$$

Note that when we insert Eq. (13) into Eq. (10), it depends on the derivatives of the scalar field. It acts as a normal scalar field potential under the conditions of slow rolling or low gradients and in the case the scalar field is near the region $M+V=0$.

In the scale invariant case, where V and U are given by $V(\phi)=f_1e^{\alpha\phi}$ and $U(\phi)=f_2e^{2\alpha\phi}$, respectively[12], it is interesting to study the shape of V_{eff} as a function of ϕ in the case when ϕ change a little bit. In this case, V_{eff} can be regarded as a scalar field potential. Then from Eq. (13) we get $R=-\kappa(V+M)$, which inserted into Eq. (10) gives,

$$V_{eff}=\frac{(f_1e^{\alpha\phi}+M)^2}{4(\epsilon\kappa^2(f_1e^{\alpha\phi}+M)^2+f_2e^{2\alpha\phi})}+\lambda. \tag{14}$$

In Fig.1 we have plotted the effective potential, V_{eff} as a function of the scalar field, ϕ for $M=-1$, $\epsilon=-1$, $f_1=1/2$, $f_2=1$, $\lambda=1/10$ and $\kappa=1$. We should mention here that the choices of the values of the different parameters was done in virtue that they respect the conditions for a stable emerging universe. Notice that it shows a flat region of positive vacuum energy for large ϕ, a minimum obtained at zero without fine tuning.

In the limit $\alpha\phi\to\infty$ the effective scalar potential, V_{eff}, becomes $V_{eff}\to\frac{f_1^2}{4(\epsilon\kappa^2f_1^2+f_2)}+\lambda$.

Notice that in all the above discussion it is fundamental that $M\neq0$. If $M=0$ the potential becomes just a flat one, $V_{eff}=\frac{f_1^2}{4(\epsilon\kappa^2f_1^2+f_2)}+\lambda$ everywhere (not only at high values of $\alpha\phi$). All the non trivial features necessary for a graceful exit, the other flat region associated to the Planck scale and the minimum at zero if $M<0$ are all lost. The case $M\neq0$ implies that we are considering a situation with S.S.B. of scale invariance.

We now want to consider the detailed analysis of emerging universe solutions. We start considering the closed Friedmann-Robertson-Walker metric,

$$ds^2=dt^2-a(t)^2\left(\frac{dr^2}{1-r^2}+r^2(d\theta^2+sin^2\theta d\phi^2)\right), \tag{15}$$

where $a(t)$ is the scale factor, t represents the cosmic time and due to homogeneity and isotropy we take the scalar field ϕ to be a function of the cosmological time only, i.e. $\phi=\phi(t)$.

We will consider a scenario where the scalar field ϕ is moving in the extreme right region, i.e. for asymptotic values of ϕ ($\phi\to\infty$). In this case, the expressions for the energy density

ρ and pressure p are given by,

$$\rho = \frac{A}{2}(1 + \kappa^2 \epsilon \dot{\phi}^2)\dot{\phi}^2 + (C + B\dot{\phi}^4), \tag{16}$$

and

$$p = \frac{A}{2}(1 + \kappa^2 \epsilon \dot{\phi}^2)\dot{\phi}^2 - (C + B\dot{\phi}^4), \tag{17}$$

where the constants A, B and C are given by

$$A = \frac{f_2}{f_2 + \kappa^2 \epsilon f_1^2}, \tag{18}$$

$$\tag{19}$$

$$B = \frac{\epsilon \kappa^2}{4(1 + \kappa^2 \epsilon f_1^2 / f_2)} = \frac{\epsilon \kappa^2}{4} A, \tag{20}$$

$$\tag{21}$$

and

$$C = \frac{f_1^2}{4 f_2 (1 + \kappa^2 \epsilon f_1^2 / f_2)} = \frac{f_1^2}{4 f_2} A - \frac{1}{4 \epsilon \kappa^2} + \lambda, \tag{22}$$

respectively.

It is interesting to notice that all terms proportional to $\dot{\phi}^4$ behave like "radiation", since

$$p_{\dot{\phi}^4} = \frac{A}{2}\kappa^2 \epsilon \dot{\phi}^4 - B\dot{\phi}^4 = (\frac{A}{2}\epsilon\kappa^2 - B)\dot{\phi}^4 = \frac{A}{4}\kappa^2 \epsilon \dot{\phi}^4, \tag{23}$$

and

$$\rho_{\dot{\phi}^4} = \frac{A}{2}\kappa^2 \epsilon \dot{\phi}^4 + B\dot{\phi}^4 = (\frac{A}{2}\epsilon\kappa^2 + B)\dot{\phi}^4 = \frac{3A}{4}\kappa^2 \epsilon \dot{\phi}^4, \tag{24}$$

so that $p_{\dot{\phi}^4} = \frac{\rho_{\dot{\phi}4}}{3}$ is satisfied. This is also a consistency check, since the ϵ terms are associated to a conformally invariant R^2 term (in the first order formalism) and in the region $\phi \to \infty$ that symmetry should hold, at least for those contributions. The ϵ terms therefore do not contribute to the trace of the energy momentum tensor in the region $\phi \to \infty$.

The equations that determines such static universe $a(t) = a_0 = constant$, $\dot{a} = 0$ and $\ddot{a} = 0$ give rise to a restriction on $\dot{\phi}_0$. Since $\ddot{a} = 0$ and it is proportional to $\rho + 3p$, we must require that $\rho + 3p = 0$, which leads to

$$(B - A\kappa^2 \epsilon)\dot{\phi}_0^4 - A\dot{\phi}_0^2 + C = 0. \tag{25}$$

This equation has two roots, the first being given by

$$\dot{\phi}_0^2 = \dot{\phi}_1^2 = \frac{-2\sqrt{f_2} - \sqrt{\Delta}}{3\sqrt{f_2}\epsilon}\,, \tag{26}$$

and the second one is

$$\dot{\phi}_0^2 = \dot{\phi}_2^2 = \frac{-2\sqrt{f_2} + \sqrt{\Delta}}{3\sqrt{f_2}\epsilon}\,, \tag{27}$$

where

$$\Delta = f_2 + 12 f_2 \lambda \epsilon + 12 \lambda f_1^2 \epsilon^2.$$

From these expression we could get some specific range of the parameters in order that either ρ_0 and $\dot{\phi_0}^2$ become positive. In the following section we will restrict these parameters in order to describe an emerging universe.

III. STABILITY OF THE STATIC SOLUTION IN THE SCALE INVARIANT R^2 MODEL

We will now consider the perturbation equations. Considering small deviations of $\dot{\phi}$ from the static emerging solution value $\dot{\phi}_0$ and also considering the perturbations of the scale factor a, we obtain, from Eq. (16), that

$$\delta\rho = A\dot{\phi}_0\delta\dot{\phi} + 4(B + \frac{1}{2}\kappa^2\epsilon A)\dot{\phi}_0^3\delta\dot{\phi}. \tag{28}$$

At the same time $\delta\rho$ can be obtained from the perturbation of the Friedmann equation $3(\frac{1}{a^2} + H^2) = \kappa\rho$, which gives under a perturbation respect to the background static solution

$$-\frac{6}{a_0^3}\delta a = \kappa\delta\rho. \tag{29}$$

On the other hand, from the second order Friedmann equation, $\frac{1+\dot{a}^2+2a\ddot{a}}{a^2} = -\kappa p$, we get that

$$\frac{2}{a_0^2} = -2\kappa p_0 = \frac{2}{3}\kappa\rho_0 = \Omega_0\kappa\rho_0, \tag{30}$$

where we have used $p_0 = -\rho_0/3$ at $a = a_0$, and we have chosen to express our result in terms of Ω_0, defined by $p_0 = (\Omega_0 - 1)\rho_0$, which for the emerging solution has the value $\Omega_0 = \frac{2}{3}$. Using this in Eq. (29), we obtain

$$\delta\rho = -\frac{3\Omega_0\rho_0}{a_0}\delta a. \tag{31}$$

Substituting this latter expression into Eq. (28) yields to a linear relation between $\delta\dot{\phi}$ and δa so that

$$\delta\dot{\phi} = D_0 \delta a, \tag{32}$$

where

$$D_0 = -\frac{3\Omega_0\rho_0}{a_0\dot{\phi}_0(A + 4(B + \frac{1}{2}A\kappa^2\epsilon\dot{\phi}_0^2))}. \tag{33}$$

Since we could write that $p = (\Omega - 1)\rho$, with

$$\Omega = 2\Big(1 - \frac{V_{eff}}{\rho}\Big), \tag{34}$$

where,

$$V_{eff} = C + B\,\dot{\phi}^4, \tag{35}$$

and therefore, the perturbation of the second order Friedmann equation leads to,

$$-\frac{2\delta a}{a_0^3} + 2\frac{\delta\ddot{a}}{a_0} = -\kappa\delta p = -\kappa\delta((\Omega - 1)\rho), \tag{36}$$

to evaluate this, we use (34), (35) and the expressions that relate the variations in a and $\dot{\phi}$ (32). Defining the "small" variable β as $a(t) = a_0(1 + \beta)$ we obtain, $2\ddot{\beta}(t) + W_0^2\beta(t) = 0\,,$ where,

$$W_0^2 = \frac{4\kappa}{3}\left[\frac{12\,B\,\dot{\phi}_0^2}{A + \dot{\phi}_0^2\,(4B + 2\kappa^2\epsilon A)} - \frac{3}{\rho_0}(C + B\dot{\phi}_0^4)\right], \tag{37}$$

with $\rho_0 = A(1 + \kappa^2\,\epsilon\,\dot{\phi}_0^2)\frac{\dot{\phi}_0^2}{2} + C + B\,\dot{\phi}_0^4$.

Stability of the static solution requires that $W_0^2 > 0$. This static solution has to respect the conditions $\dot{\phi}_0^2 > 0$ and $\rho_0 > 0$. Only the second solution, i.e. $\dot{\phi}_2^2$ (see Eq. (27)), fulfills these requirements, providing that the parameters satisfy the following inequalities

$$\epsilon \; < \; 0 \tag{38}$$

$$-\epsilon f_1^2 \; < \; f_2 \tag{39}$$

$$0 < \; \lambda \; < -\frac{1}{12\epsilon\left(1 + \frac{\epsilon f_1^2}{f_2}\right)}, \tag{40}$$

Here, we have taken $\kappa = 1$.

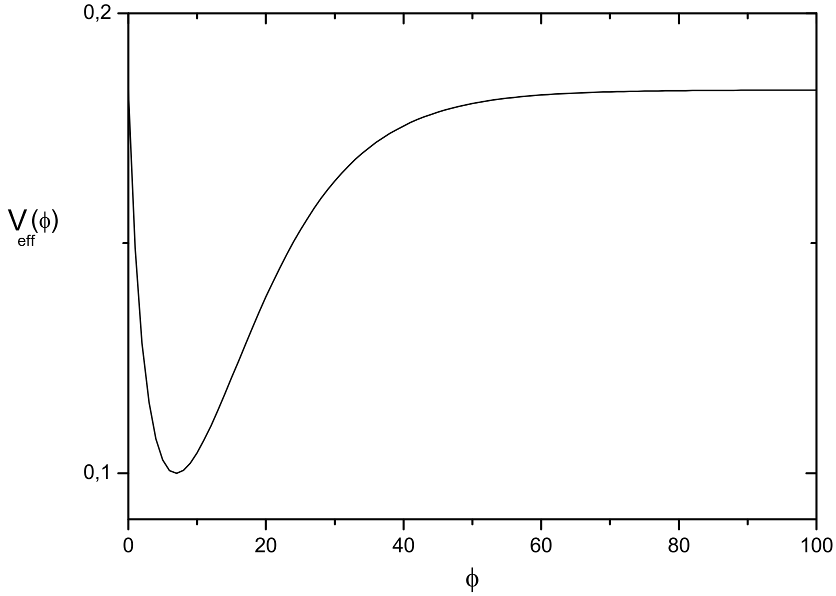

FIG. 1: The form of effective potential $V_{eff}(\phi)$ versus the scalar field ϕ. We have used $M=-1$, $\epsilon=-1$, $f_1=1/2$, $f_2=1$, $\lambda=1/10$ and $\kappa=1$.

IV. STABILITY OF THE STATIC SOLUTION: DYNAMICAL SYSTEM.

The study of the stability of the static solution and the properties of the different equilibrium points could be done in a more systematic way by using a dynamical system approach. In this scheme we rewrite the Friedmann and the conservation of energy equations as an autonomous system in terms of the variables H and $x\equiv\dot{\phi}^2$. In order to do so, we differentiating the Fridmann equation and after using the expressions for ρ and p, Eq. (16, 17), and the energy conservation equation we obtain:

$$\dot{H}=\frac{1}{a^2}-\frac{\kappa}{2}\,A\,(1+\kappa^2\,\epsilon\,x)x\,. \tag{41}$$

We can rewrite Eq. (41) and the energy conservation equation as the following two equations.

$$\dot{H} = \frac{\kappa}{3}\Big[C + B\,x^2 - A(1+\kappa^2\,\epsilon\,x)\,x\Big] - H^2, \tag{42}$$

$$\dot{x} = -\frac{3A\,(1+\kappa^2\,\epsilon\,x)\,x}{\frac{A}{2} + A\,\kappa^2\,\epsilon\,x + 2B\,x}\,H\ , \tag{43}$$

where we have eliminated a from Eq. (41) by using the Friedmann equation and we have used the definition of the variable x in order to get Eq. (43) from the energy conservation equation. The equations (42) and (43) are a two-dimensional autonomous system on the variables H and x.

In order to study the stability of the static solutions we look for critical points of the system (42) and (43). These points are

$$\Big\{H = 0,\ x = \frac{A - \sqrt{A^2 + 4(A\epsilon - B)C}}{2(B - A\epsilon)}\Big\}; \tag{44}$$

$$\Big\{H = 0,\ x = \frac{A + \sqrt{A^2 + 4(A\epsilon - B)C}}{2(B - A\epsilon)}\Big\}; \tag{45}$$

$$\Big\{H = -\sqrt{\frac{C}{3}},\ x = 0\Big\}; \Big\{H = \sqrt{\frac{C}{3}},\ x = 0\Big\}\ ;\ \Big\{H = -\sqrt{\frac{\frac{B}{\epsilon^2} + C}{3}},\ x = -\frac{1}{\epsilon}\Big\}; \tag{46}$$

$$\Big\{H = \sqrt{\frac{\frac{B}{\epsilon^2} + C}{3}},\ x = -\frac{1}{\epsilon}\Big\}. \tag{47}$$

The critical points have different properties depending on the values of the parameters of the model $(f_1, f_2, \epsilon,\ \lambda)$. At this moment we are not going to give an exhaustive description of these properties for all the critical points, instead, we are going to focus on the particular critical points which are related with static universe. From the definition of the variables H and x we can note that only the first two critical points Eqs. (44, 45) correspond to a static universe. In order to study the nature of these two critical points we linearize the equations (42) and (43) near these critical points. From the study of the eigenvalues of the system we found that the first critical point, Eq. (44), could be a center or a saddle point, depending on the values of the parameters of the model. On the other hand the second critical point Eq. (45) is a saddle.

Stable static solutions correspond to a center. Then, by impose the critical point, Eq.(44), becomes a center, we recover the stability condition Eqs.(38), (39) and (40).

In order to satisfy the requirements of stability we are going take the values $\epsilon = -1$, $f_1 = 1/2$, $f_2 = 1$ and $\lambda = 1/10$.

In Fig. 2 it is shown a phase portrait for four numerical solution to Eqs. (42) and (43). Also, in this figure we have included the *Direction Field* of the system in order to have a picture of what a general solution look like. In this figure are four of the six critical points described in Eqs. (44)-(47). One of this point is the center equilibrium point ($H = 0, x = 0.56$), the saddle point ($H = 0, x = 0.77$), the point ($H = 0.18, x = 1$) is a future attractor and ($H = -0.18, x = 1$) is a past attractor. The other equilibrium points are ($H = 0.38, x = 0$) and ($H = -0.38, x = 0$) which are a future attractor point and a past attractor point respectively.

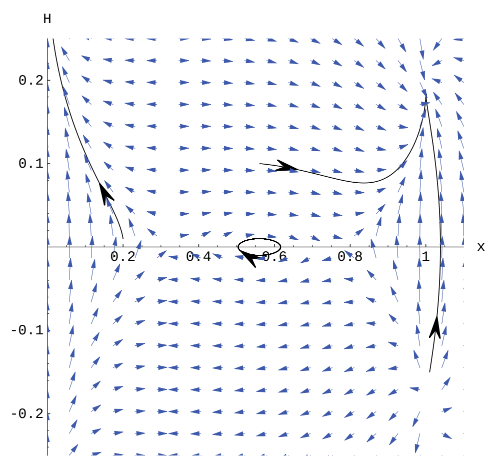

FIG. 2: Plot showing part of the *Direction Field* of the system and four numerical solutions.

As we have mentioned, only critical points with $H = 0$ represent a static universe. Then in our case only the critical point which is a center correspond to a solution which represent a static and stable universe. In Fig. 3 we show the *Direction Field* near this critical point ($H = 0, x = 0.56$) together with two numerical solution.

V. INFLATION AND ITS GRACEFUL EXIT

The emerging phase owes its existence to a strictly constant vacuum energy (which here is represented by the value of A) at very large values of the field ϕ. In fact, while for $M = 0$

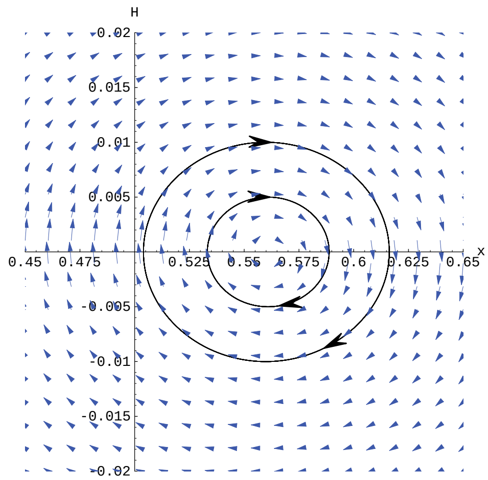

FIG. 3: Plot showing the *Direction Field* near the center critical point and two numerical solution.

the effective potential of the scalar field is perfectly flat, for any $M \neq 0$ the effective potential acquires a non trivial shape.

We consider then the relevant equations for the model in the slow roll regime, i.e. for $\dot{\phi}$ small and when the scalar field ϕ is large, but finite and we consider the first corrections to the flatness to the effective potential. Dropping higher powers of $\dot{\phi}$ in the contributions for the kinetic energy and in the scalar curvature R, we obtain

$$\rho = \frac{1}{2}\gamma\dot{\phi}^2 + V_{eff}, \tag{48}$$

$$\gamma = \frac{\chi}{\chi - 2\kappa\epsilon R}, \tag{49}$$

$$R = -\kappa(V + M). \tag{50}$$

Here, as usual $\chi = \frac{2U(\phi)}{M+V(\phi)}$. In the slow roll approximation, we can drop the second derivative term of ϕ and the second power of $\dot{\phi}$ in the equation for H^2 and we get

$$3H\gamma\dot{\phi} = -V'_{eff}, \tag{51}$$

$$3H^2 = \kappa V_{eff}, \tag{52}$$

where $V'_{eff} = \frac{dV_{eff}}{d\phi}$. The relevant expression for V_{eff} will be that given by (14), i.e., where all higher derivatives are ignored in the potential, consistent with the slow roll approximation.

We now display the relevant expressions for the region of very large, but not infinite ϕ,

these are:

$$V_{eff} = C + C_1 exp(-\alpha\phi), \tag{53}$$

$$\chi = 2\frac{f_2}{f_1}exp(\alpha\phi) - 2M\frac{f_2}{f_1^2}, \tag{54}$$

and

$$\gamma = \gamma_0 + \gamma_1 exp(-\alpha\phi). \tag{55}$$

The relevant constants that will affect our results are, C, as given by (22) and C_1 and γ_0 given by

$$C_1 = -\frac{8\epsilon\kappa^2 f_1^3 M}{(4f_2 + 4\kappa^2\epsilon f_1^2)^2} + \frac{2f_1 M}{4f_2 + 4\kappa^2\epsilon f_1^2}, \tag{56}$$

and

$$\gamma_0 = \frac{f_2}{f_2 + \kappa^2\epsilon f_1^2}, \tag{57}$$

respectively.

Using Eq. (53) we can calculates the key landmarks of the inflationary history: first, the value of the scalar field where inflation ends, ϕ_{end} and a value for the scalar field ϕ_* bigger than this ($\phi_* > \phi_{end}$) and which happens earlier, which represents the "horizon crossing point". We must demand then that a typical number of e-foldings, like $N = 60$, takes place between ϕ_*, until the end of inflation at $\phi = \phi_{end}$.

To determine the end of inflation, we consider the quantity $\delta = -\frac{\dot{H}}{H^2}$ and consider the point in the evolution of the Universe where $\delta = 1$, only when $\delta < 1$, we have an accelerating Universe, so the point $\delta = 1$ represents indeed the end of inflation. Calculating the derivative with respect to cosmic time of the Hubble expansion using (52) and (51), we obtain that the condition $\delta = 1$ gives

$$\delta = \frac{1}{2\gamma}(V'_{eff}/V_{eff})^2 = 1, \tag{58}$$

working to leading order, setting $\gamma = \gamma_0$, $V_{eff} = C$ and $V'_{eff} = -\alpha C_1 exp(-\alpha\phi_{end})$, this gives as a solution,

$$exp(\alpha\phi_{end}) = -\frac{\alpha C_1}{C\sqrt{2\kappa\gamma_0}}, \tag{59}$$

notice that if M and f_1 have different signs and if $\epsilon < 0$, $C_1 < 0$ for the allowed range of parameters the stable emerging solution, so $-C_1$ represents the absolute value of C_1. We now consider ϕ_* and the requirement that this precedes ϕ_{end} by N e-foldings,

$$N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi = -\int \frac{3H^2\gamma}{V'_{eff}} d\phi, \tag{60}$$

where in the last step we have used the slow roll equation of motion for the scalar field (51) to solve for $\dot{\phi}$. Solving H^2 in terms of V_{eff} using (52), working to leading order, setting $\gamma = \gamma_0$ and integrating, we obtain the relation between ϕ_* and ϕ_{end},

$$exp(\alpha\phi_*) = exp(\alpha\phi_{end}) - \frac{N\alpha^2 C_1}{C\kappa\gamma_0}, \tag{61}$$

as we mentioned before $C_1 < 0$ for the allowed range of parameters the stable emerging solution, so that $\phi_* > \phi_{end}$ as it should be for everything to make sense. Introducing Eq. (59) into Eq. (61), we obtain,

$$exp(\alpha\phi_*) = -\frac{C_1}{C\sqrt{\kappa}}\left(\frac{\alpha}{\sqrt{2\gamma_0}} + \frac{N\alpha^2}{\sqrt{\kappa}\gamma_0}\right). \tag{62}$$

We finally calculate the power of the primordial scalar perturbations. If the scalar field ϕ had a canonically normalized kinetic term, the spectrum of the primordial perturbations will be given by the equation

$$\frac{\delta\rho}{\rho} \propto \frac{H^2}{\dot{\phi}}, \tag{63}$$

however, as we can see from (48), the kinetic term is not canonically normalized because of the factor γ in that equation.

In this point we will study the scalar and tensor perturbations for our model where the kinetic term is not canonically normalized. The general expression for the perturbed metric about the Friedmann-Robertson-Walker is

$$ds^2 = -(1+2F)dt^2 + 2a(t)D_{,i}dx^i dt + a^2(t)[(1-2\psi)\delta_{ij} + 2E_{,i,j} + 2h_{ij}]dx^i dx^j,$$

where F, D, ψ and E are the scalar type metric perturbations and h_{ij} characterizes the transverse-traceless tensor perturbation. The power spectrum of the curvature perturbation in the slow-roll approximation for a not-canonically kinetic term becomes Ref.[17](see also Refs.[18])

$$P_S = k_1 \left(\frac{\delta\rho}{\rho}\right)^2 = k_1 \frac{H^2}{c_s\,\delta}, \tag{64}$$

where it was defined "speed of sound", c_s, as

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}},$$

with $P(X, \phi)$ an function of the scalar field and of the kinetic term $X = -(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Here $P_{,X}$ denote the derivative with respect X. In our case $P(X, \phi) = \gamma(\phi)\, X - V_{eff}$, with $X = \dot{\phi}^2/2$. Thus, from eq.(64) we get

$$P_S = k_1 \frac{H^4}{\gamma(\phi)\dot{\phi}^2}. \tag{65}$$

The scalar spectral index n_s, is defined by

$$n_s - 1 = \frac{d \ln P_S}{d \ln k} = -2\delta - \eta - s, \tag{66}$$

where $\eta = \frac{\dot{\delta}}{\delta\, H}$ and $s = \frac{\dot{c_s}}{c_s\, H}$, respectively.

On the other hand, the generation of tensor perturbations during inflation would produce gravitational wave. The amplitude of tensor perturbations was evaluated in Ref.[17], where

$$P_T = \frac{2}{3\pi^2} \left(\frac{2XP_{,X} - P}{M_{Planck}^4} \right),$$

and the tensor spectral index n_T, becomes

$$n_T = \frac{d \ln P_T}{d \ln k} = -2\delta,$$

and they satisfy a generalized consistency relation

$$r = \frac{P_T}{P_S} = -8\, c_s\, n_T. \tag{67}$$

Therefore, the scalar field (to leading order) that should appear in Eq. (63) should be $\sqrt{\gamma_0}\phi$ and instead of Eq. (65) , we must use

$$\frac{\delta\rho}{\rho} = \frac{H^2}{\sqrt{\gamma_0}\dot{\phi}}. \tag{68}$$

The power spectrum of the perturbations goes, up to a factor of order one, which we will denote k_1 as $(\delta\rho/\rho)^2$, so we have,

$$P_S = k_1 \left(\frac{\delta\rho}{\rho} \right)^2 = k_1 \frac{H^4}{\gamma_0 \dot{\phi}^2}, \tag{69}$$

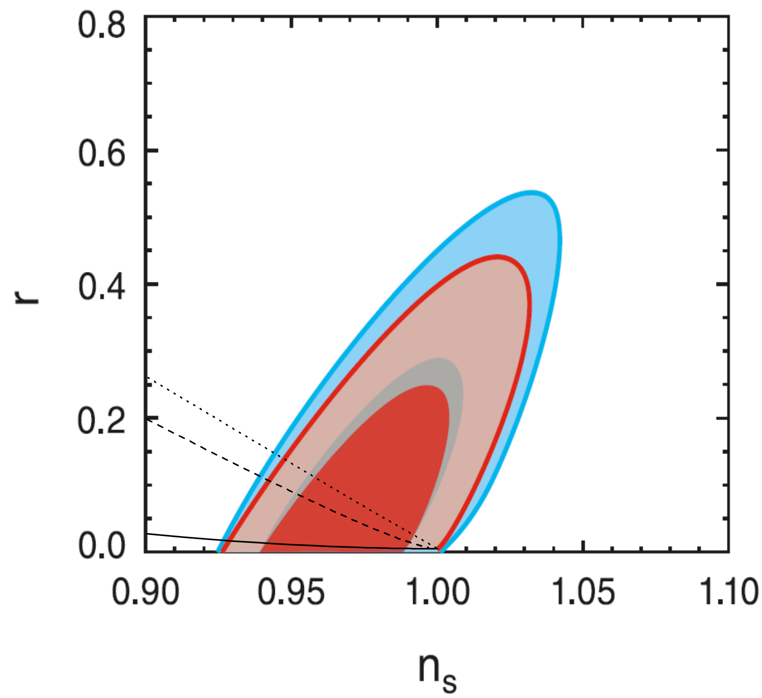

FIG. 4: The plot shows r versus n_s for three values of α. For $\alpha = 1$ solid line, $\alpha = 0.1$ dash line and $\alpha = 0.01$ dots line, respectively. Here, we have fixed the values $M = -1$, $\epsilon = -1$, $f_1 = 1/2$, $f_2 = 1$, $\lambda = 1/10$ and $\kappa = 1$, respectively. The seven-year WMAP data places stronger limits on the tensor-scalar ratio (shown in red) than five-year data (blue) [19].

this quantity should be evaluated at $\phi = \phi_*$ given by (62). Solving for $\dot{\phi}$ from the slow roll equation (51), evaluating the derivative of the effective potential using (53) and solving for H from (52), we obtain, to leading order,

$$P_S = k_1 \frac{\kappa^3 \gamma_0 C^3}{3\alpha^2 C_1} exp(2\alpha\phi_*), \tag{70}$$

using then (62) for $\exp(\alpha\phi_*)$, we obtain our final result,

$$P_S = k_1 \frac{\kappa^2 C}{3} (\frac{1}{\sqrt{2}} + \frac{N\alpha}{\sqrt{\gamma_0 \kappa}})^2, \tag{71}$$

it is very interesting first of all that C_1 dependence has dropped out and with it all dependence on M. In fact this can be regarded as a non trivial consistency check of our estimates, since apart from its sign, the value M should not affect the results. This is due to the fact

that from a different value of M (although with the same sign), we can recover the original potential by performing a shift of the scalar field ϕ.

In Fig.4 we show the dependence of the tensor-scalar ratio r on the spectral index n_s. From left to right $\alpha = 1$ (solid line), $\alpha = 0.1$ (dash line) and $\alpha = 0.01$ (dots line), respectively. From Ref.[19], two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r and n_s, the spectral index of fluctuations, defined at $k_0 = 0.002$ Mpc^{-1}. The seven-year WMAP data places stronger limits on r (shown in red) than five-year data (blue)[20]. In order to write down values that relate n_s and r, we used Eqs.(66) and (67). Also we have used the values $M = -1$, $\epsilon = -1$, $f_1 = 1/2$, $f_2 = 1$, $\lambda = 1/10$ and $\kappa = 1$, respectively.

From Eqs.(60), (66) and (67), we observed numerically that for $\alpha = 1$, the curve $r = r(n_s)$ (see Fig.4) for WMAP 7-years enters the 95% confidence region where the ratio $r \simeq 0.011$, which corresponds to the number of e-folds, $N \simeq 32$. For $\alpha = 0.1$, $r \simeq 0.103$ corresponds to $N \simeq 227$ and for $\alpha = 0.01$, $r \simeq 0.136$ corresponds to $N \simeq 14137$. From 68% confidence region for $\alpha = 1$, $r \simeq 0.010$, which corresponds to $N \simeq 34$. For $\alpha = 0.1$, $r \simeq 0.08$ corresponds to $N \simeq 240$ and for $\alpha = 0.01$, $r \simeq 0.109$ corresponds to $N \simeq 14279$. We noted that the parameter α, which lies in the range $1 > \alpha > 0$, the model is well supported by the data as could be seen from Fig.4.

VI. DISCUSSION AND CONCLUSIONS

The consideration of a scale invariant action leads to exactly the type of scalar field potentials needed to obtain an emerging universe scenario. The addition of a R^2-term gives rise to solutions where $\epsilon < 0$, $-\epsilon f_1^2 < f_2$ and $0 < \lambda < -\frac{1}{12\epsilon\left(1+\frac{\epsilon f_1^2}{f_2}\right)}$ and to the stability of the initial Einstein universe solution.

We find that this is accomplished because the ϵR^2 term is shown to have the effect of introducing nonlinearities in the theory that stabilize the solutions.

In addition in order for the whole idea to work we need to take $M \neq 0$, which means we are considering the S.S.B. of the theory and this is what gives the effective potential a non trivial shape, which allows a graceful exit from the inflationary phase and can give the adequate power of the density perturbations.

It should be pointed out also that the R^2 theory studied here, in the context of an action

that contains a measure Φ independent of the metric and with the use of the first order formalism (the connections are considered as independent degrees of freedom in the action principle), leads to equations of motion that are only second order, i.e. only second derivatives of the metric and the matter fields appear (although higher powers of the derivatives of the scalar field do appear). This after the new measure Φ and the connections are solved (through the equations of motion) in terms of the metric and matter fields and after we express our results in the Einstein frame.

This is in contrast to the usual R^2 theories in the second order formalism and with standard measure everywhere in the action. In this case, these theories lead to fourth order equations for the metric field. In Ref.[21] it was shown that the fourth order structure of the equations can be reformulated as a system of second order equations which contain however one additional degree of freedom, a scalar field. For studies of inflation in the usual R^2 theories in the second order formalism, see Ref.[22]. In the case of the usual R^2 theories this scalar field contains a potential with a flat region, as shown in Fig.1.

Using the WMAP seven-year data, we have found some constraints for the parameters appearing in our model. In particular, Fig.4 shows that for the values of the parameter $1 > \alpha > 0$, the model is well supported by the WMAP data.

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